Pilot Experiment:  
The Effect of the Taxation of Risky Income on Investment Behavior

This experiment is in a very early stage. I am presenting it at this stage both because I think your comments would be very helpful in planning the experiment and also because I think there may be some interest in the results even at this early stage, although as you will see the conclusions one can draw from the data so far are quite modest. The analysis here is of raw pilot data, which is not intended for publication. The upside is that it also contains subject descriptions of what is going on, which would not usually make it into a published study.

I apologize for the data presentation as well. So far the experiment is a fairly low-tech affair. This is so far being done on an excel spreadsheet, and the production values at this point are low (think of it as the storyboard version).

Again, I apologize for the early stage of this, but I think it is an important project, and even the preliminary data might be of some interest.

I. The Model

A standard model for the behavior of investors under taxation of risky income was developed by Evsey Domar and Richard Musgrave (Proportional Taxation and Risk-Taking 58 Q. J. ECON. 388, 1944). It was further developed by Joseph Stiglitz in article in QJE in 1969 (The Effects of Income, Wealth, and Capital Gains Taxes on Risk-Taking), and is a staple of the literature about the effects of income taxes. (See Myles, PUBLIC ECONOMICS, 1995, Ch.7) This model predicts that under a symmetrical income tax (discussed more fully below), and if a variety of conditions are
met, investors be able to will shift their portfolios so that the effect on the investor will be the same is if there was no tax on risky portion of their returns. That is, an income tax will not burden the risky return on capital investment. One of the predictions of the model is that investors will increase the percentage of the portfolio allocated to risky securities as opposed to riskless securities. As discussed below, a key limitation of this model is the strength of the conditions that have to be met in order for it to apply.

To illustrate how this model works, let us use a highly stylized model of investment behavior. Suppose that following most standard models of investment, each investor has a choice of two investments, one a safe, riskless investment and the other a risky investment with a higher expected value. The problem for investors is to allocate their portfolios between these two investments.\(^1\) The investor's returns from a particular portfolio allocation will be \(W_1 = (1-x)W_0 + xW_0r\), where \(W_0\) is the level of wealth in period 0 and \(W_1\) is the wealth in period 1, \(x\) is the percentage of the portfolio invested in the risky asset, and \(r\) is the stochastic return to the risky asset. Notice that under this simple version of this model, there is no return on the riskless investment. In addition, we assume that the distribution of \(r\) is known to the investor. It can be either a discrete distribution (e.g., 50% chance of a $10 loss and a 50% chance of a $15 gain) or a continuous distribution. Standard investment models assume that individual investors will select \(x\) so as to optimize their expected utility.

If we introduce taxation of income on the risky asset (with full deduction allowed for losses) at the rate \(t\), where \(0 < t < 1\), the final random wealth is

\[
W_1 = (1-x)W_0 + xW_0r(1-t).\]

One can show that the first order condition for an interior optimum is

\[
\int_{r_-}^{r_+} (rW_0 + (1-t)x_0)f(r)dr = 0
\]

where \(f(r)\) is the probability density function, and \(r_-\) and \(r_+\) are the limits of the possible returns.

One consequence of this model is that, if \(t\) changes, the optimum \(x\) changes to keep \((1-t)x\) constant. This occurs because, in order to stay at a utility maximum, we must keep \(E[u'(W)] = 0\) and therefore, if \(t\) changes, \(x\) has to change so that \(x_0/(1-t) = x_1\), where \(x_0\) was the allocation to the risky asset without the tax, and \(x_1\) is allocation with the tax. If the tax rate on risky income increases, so too does the amount of wealth held in the risky asset. If only the risky rate of return is taxed, then taxing the income from capital at a higher rate would encourage more investment in the taxed asset, rather than less. Therefore, imposing a tax on risky returns should result in more capital being allocated to risky assets. Furthermore, as pointed out by Agnar Sandmo, (Differential Taxation and the Encouragement of Risk-Taking, Economics Letters, 1989) if there are two risky assets whose returns are i.i.d (independent and identically distributed)
and the incomes of each are taxed at two different rates, then the more highly taxed asset will have a greater allocation of the portfolio than the lower taxed asset.

The intuition of the model is that, by taking a portion of the risky return, both positive and negative, the government is effectively becoming a partner in the risky investment, with a percentage interest of \((x/(1-t)) - x\). Because of this, the tax does not only reduce the return on the investment, it also reduces its risk. By shifting more capital to the risky asset, taxpayers are able to have the same risk return trade-off as without the tax.

This model is central to a number of standard results in public finance. For example, it is the basis for arguing that the difference between consumption taxes and income taxes is a tax on riskless return on investment. (Al Warren, *How Much Capital Income Taxed Under an Income Tax is Exempt Under a Cash Flow Tax*, Tax Law Review, 1996). It is also the basis for the argument about the economic effects of wealth taxes as compared to income and consumption taxes (Stiglitz, QJE 1969). This literature has been review in many places including, Gareth Myles, *PUBLIC ECONOMICS*, 1995 Ch. 7 and James Poterba, *Taxation, Risk-taking and Household Portfolio Behavior* 3 Handbook of Public Economics 1109 (2002). This model is also important in a number of recent articles on the effects of taxation on risk-taking. This model has also been important in discussions of the second layer of tax on corporate income as well as the taxation of derivatives and other financial instruments. Because of the growing interest in the use of the model to analyze investment decisions, understanding whether this is a reasonable representation of how investors in fact make their choices would be an important addition to the literature.

One of the key limitations of this model, as commonly presented, is that it is only a partial equilibrium analysis. That is, it only looks at the effects of taxes on investor behavior, without examining other changes that result from the imposition of the taxes. It assumes that the taxes themselves essentially perform no useful function. However, if the taxes are used to fund public goods that the individuals values then they have not in fact reduced the risk they face, but rather they have simply transferred the risk from private goods into public goods. This is one of the most serious problems of the application of the Domar-Musgrave model. The models do not give unambiguous predictions for how investors will behave in a situation where not only the risk is transferred to a public good, but it is also the case that prices of the various investments will shift depending on the demand for the various investments. That is, once one changes the total level of risk to which the investors are subject, it is no longer clear that the predicted shifts will occur.

II. EXPERIMENT

This experiment is designed to test this model. This theory is very difficult to test with econometric data. While there has been some econometric work which might be argued supports this model, in all of this work there are too many confounding factors for any of them to be a reasonable test of the theory. In fact, it is highly unlikely that econometric data, given the current tools we have for data analysis, will be able resolve the question of whether this
model accurately describes investor behavior. One key problem with testing this model is that it assumes that the tax system allows for what are referred to as full loss offsets. This means that if you earn income, you would pay a tax equal to your tax rate multiplied by your income (or \( tx \) in the model, yielding an after tax return of \( (1-t)x \)) and if you lose money, you get a check from the government equal to your losses multiplied by your tax rate (\( tx \) in the model yielding an after tax loss of \( (1-t)x \)). This makes the income tax symmetrical in gains and losses. This is sometimes referred to as a perfect or normative income tax. The actual tax system is not perfectly symmetrical, because there are significant restrictions on the use of losses. These are in place in large measure to prevent abuse of the tax system due to the realization doctrine. Furthermore, because tax legislation almost invariably contains many complicated provisions which affect the tax rate and loss offset provisions is complicated ways, it is very difficult to test this empirically.

Stiglitz showed that even without full loss offsets, it may be possible to still get some of this effect, but then it is an empirical question, because it depends on the risk aversion of the investors. This is important as a policy matter because if this effect does occur, it could mean that taxes could be structured to have lower deadweight losses (and actually lower losses over all) (See Terrence Chorvat, Apologia for the Double Taxation of Corporate Income, 38 Wake Forest L. Rev. 239 (2003)). Even if one could overcome the loss offset problem, econometric data might not conclusively test the model because of the enormous complexity of the tax system, the significant amount of heterogeneity that one finds among taxpayers, not only in risk aversion, but also in tax position and the fact that it is never the case that only one change to the tax rules occurs in isolation. The purpose of this experiment is to test the model at a point when its predictions are unambiguous.

In order for the model to yield unambiguous predictions, a number of relatively stringent conditions have to be met. As discussed, the tax system must be symmetrical between gains and losses. In addition, the rate of return on a riskless asset would have to be zero. Even though small rates of return on riskless investment will likely still yield shifts into the risky asset, the predictions become more ambiguous the higher the risk-free rate of return, and the longer the investment time horizon of the investor. In addition, some of the predictions of the model only operate where the marginal rate of return on the risky investment is constant. If the rate of return declines with the amount invested in the risky investment, this can affect how the investor is willing to invest, and so the predictions of the model become dependent both on the exact shape of the utility curves of the investors and the rate of decline of the returns to investors.

A simple way to test this model is to give subjects a stake or portfolio and have them allocate their portfolio between two assets, a risky asset and a riskless asset. One would need to make the returns to risky assets random, in a way that the subjects understand that these returns are random and not guessable. One way to do this would be to use some commonly understood randomizing device. The easiest return distribution to understand and to operate would be where the distribution of returns on the risky asset was discrete (e.g., a 50% chance of a $10 loss and 50% chance of $10...
$15 gain). In this case, the randomization could be accomplished by flipping a coin, or roll a die, etc. In the experiment as conducted so far, I have chosen to chose between two results by the flip of a coin. We might attempt to test the model with a continuous distribution of the returns by having it derived from a random number generator on a computer, although there may be more suspicion or confusion over the distribution in this case. In the experiment, there were two investment conditions prior to the experimental treatments in order for the subjects to get used to the experiment and so that they might feel some ownership over the points.

This experiment utilizes a within-subjects design, for a number of reasons. First, because the theory is effectively a within-subjects theory (that is, the model predicts how each individual investor will behave), this design seems appropriate. Second, because the subject pool is relatively small, and may stay that way for the experiment proper, a within subjects design yields more power. Of course, any time that one uses a within-subjects design, a concern arises about the order of the questions posed, and if the order they are posed will make a difference in the behavior. One could argue that if we have enough subjects that if they are randomly allocated between the two treatments we should be able to make a comparison between the two treatments. In addition, for policy purposes we are interested in the societal reaction, not just individual reactions. If there are a sufficient number of individuals, we should be able to run the experiment as a between subjects experiment, in which case we should still be able to see some difference. The data here suggest that given the huge degree of heterogeneity of investment responses observed, it would seem that the number of subjects necessary to have any power would quite large.

The prediction of the Domar-Musgrave model in situations where all of the conditions are met is that investors should shift the assets so that they will now allocate \( x/(1-t) \) to the risky asset. Let us define \( z_i \) as the percentage of the portfolio allocated to the risky asset by investor i after the tax has been imposed, and let us also define \( s_i = (z_i - x_i)/x_i \). If a tax on the returns to the risky asset increase the capital allocated to the risky asset, then \( \mu_s > 0 \). If the Domar-Musgrave model is accurate, \( \mu_s = 1/(1-t) - 1 \). Therefore, this experiment is designed to test what effect imposing a tax has on the allocation of assets between a riskless and risky investment, and secondarily to investigate whether the Domar-Musgrave model is an accurate depiction of the results.

One might expect that there will be a number of subjects who will not follow the model's predictions. So that \( \Sigma |s| \) will almost certainly be greater than zero. In fact, the data show that \( \Sigma |s| \) is substantially greater than zero.

This experimental design is clearly in the very early stages. There are probably a nearly infinite number of additional ways the data could be analyzed and I would most certainly welcome suggestions on this or any other aspect of the experiment.

QUICK SUMMARY OF THE EXPERIMENT:
1. Test the portfolio allocations without any tax. Here, we would determine the value of $x_i$ for each subject.

2. Test the portfolio allocations with a tax. Here, we would determine the value of $z_i$ for each subject.

3. Analyze the difference between the changes predicted by the model to determine the effect of taxes on the willingness to invest in a risky asset. We would first like to know whether the tax increases or decreases the amount allocated to the risky asset, and secondarily whether the Domar-Musgrave model is accurate in its predictions.

Data:

The subjects for this experiment were recruited from my Partnership Tax class. I obtained consent to be a subject in an experiment from each of the students, as required by the George Mason IRB. I have 16 students enrolled in the class and they all agreed to participate in the experiment. The responding students were 10 males and 6 females, and were all tax students who had the basic federal income tax course prior to the class. In addition, all of the subjects had had the required class on economic analysis for the law which includes notions such as expected value and risk. The incentive given in this experiment were extra credit points on the exam, which can be done in this case because the class is not subject to the standard mandatory curve of the law school.

The sample size is both rather small and also somewhat idiosyncratic. All of the subjects have demonstrated at least some interest in tax law (by taking at least two tax classes). It is certainly arguable that the results found in this group cannot be extended to other law students or even investors generally. There are also many critiques that one can make for arguing that extra credits points are different than money (cite the study where that was done before.)

The methods used in the experiment are of a "pencil and paper" variety rather than conducted on the computer, even though the latter is becoming somewhat standard for economic experiments. The instructions and experimental treatments are attached. This was done because this certainly affords the student a way to understand that the returns are truly random and that there was no way to cheat them.

They were awarded with 100 extra credit points just for agreeing to participate in the experiment. The final exam will be worth 600 points. There have been 4 four rounds of investments so far. In the all rounds of the experiment so far, the students were expected to make a choice between two investment options: a risky and a riskless choice. The riskless choice was always option #1, and the risky choice was always option #2. The first two rounds were intended to get the subjects used to making investment decisions, and the way in which the experiment worked.

The two rounds which are of interest to the experiment for this question is how they treated treatment in round 3 and in round 4.

The results of the selection are given on the attached spreadsheet. The first column gives the anonymous
student numbers for the experiment. The next four columns (columns 2-5) give the percentage of the portfolio that was allocated to the risky asset in each of the four treatments. The analysis focuses on the last two treatments. As can be seen by examining the questions, the portfolio choice in numbers 3 and 4 is the same except that in number 4 there is symmetrical 30% tax imposed on the returns on the investment. The sixth column gives the change in the percentage allocation between Treatments 3 and 4. The seventh column gives the percentage increase in the amount allocated to the risky asset as a result of the tax. At the bottom of the column are stated the average increase, the standard deviation of the increase, and the standard error. The Domar-Musgrave model would predict that because there has been a 30% tax imposed, then the subjects should increase the amount allocated to the risky investments by \(\frac{1}{1-0.3} - 1\) or 42.857%.

III. Analysis of the Results

Qualitative analysis- One of the key predictions of the model is that imposing an income tax will result in an increase in the allocation to the risky assets. A number of subjects chose to allocate all of their investments to the risky investment under all circumstances. Here it is clear, that given the fact that they have chosen a corner solution, it was not possible for them to increase their investment in the risky asset. It is not clear whether the Domar-Musgrave model does not apply to these investors, because if an appropriate level of risk can be found for them to allocate less than all to the risky investment. They may have allocated less to the risky asset in the tax-free condition. However, when asked why they chose the allocation they did (as all subjects were), they responded that they calculated the expected value of the investment. If this is true, these subjects acted as risk-neutral investors, so would always invest in the investment with the highest return. The predictions of the Domar-Musgrave do not apply well to such investors. Therefore, it is not entirely clear that their data should necessarily not be counted against the model. There were also a fair number of subjects who increased their allocation to the risky asset, while there were a fair number of others who decreased their investment.

Unfortunately, given the data, there is little that can be said with statistical significance. The average increase in the amount of the risky investment was 17.93%, with a standard error of 18.676%. Given the number of observations (16), using a two tailed \(t\)-test, the 95% confidence interval for this is \(-21.87 < \mu_s < 57.73\). Because of the width of this confidence interval, one cannot eliminate the possibility that the Domar-Musgrave model is accurate on a societal basis. In fact, one cannot even state that the tax necessarily decreases investment in the risky asset.\(^\text{11}\) Therefore, one can cannot conclude directly very much about the reaction from these results.

Another interesting way to look at the data is the aggregated total percentage change in the amount allocated to the risky asset. This is important because if there is substantial heterogeneity in the population, and those who allocate
a higher percentage of their assets in the tax free portfolio are more likely to allocate away from risky assets when the tax is imposed, then on a societal basis, even if most individuals increase the allocation, as a society we may actually decrease the allocation to risky assets. The average after imposing a tax was actually a decrease of 2.8 percentage points in the amount allocated to the risky assets. Because this result is also not statistically significant, we cannot conclude that there necessarily will be a decrease in the risky assets, it was apparently the case that those who allocated the most to the risky asset in the taxfree case were more likely to decrease the allocation in the taxable case.

In addition, there are variety of ways in which this experiment departs from real investment choices. Here, there are confronted with the problem in a context where consistency may seem more important. One that these results may make one think about is that the comparative static prediction gave the right sign of the investment shift, but does appear to have gotten the wrong value for the investment level.

The data show a large degree of heterogeneity among subjects. Questioning afterwards, done anonymously by student number, indicates that the subjects were aware that there was a negative tax if there was a loss (even those who decreased their allocation to the risky asset seemed to understand this). However, there was a lot of confusion over the proper role this effect on investment allocation. Specifically, a number of subjects said they didn't know how to include it in their calculations.

There are a number of other questions which I intend on pursuing along this line. One of the most interesting questions is what happens in the general equilibrium situation, where the risk is returned to the participants via a public good. Will there still be a greater allocation to the risky asset or will it still have the same allocation. Another question is what would be the response of those who opted for 100% allocation to the risky asset in all situations to an ability to borrow money to invest it in the risky asset.

Conclusion

What can we conclude from this data? First of all, it is quite clear that the behavioral responses, at least initially to the imposition of a tax are quite heterogenous. At least 6 of the subjects actually decreased the amount they allocated to the risky asset as a result of the imposition of the tax (This was therefore the modal response). Five of the subjects made no change, but 3 of them had already chosen to invest 100% of their portfolio in the risky asset. Only five of the subjects increased their investment in the risky asset.

While this pilot does not show that one can reject the Domar-Musgrave model, it does give one pause about using such a model. It is possible that the model does reflect the investment choices of some of the population, but it may not represent a large segment of the population. Such a finding is extremely common in experimental economics. Therefore, at a minimum this pilot study indicates that further research needs to be done to determine how individuals make these allocations, and how we might be able to understand this heterogeneity.
Assignment 1

As a result of agreeing to do this assignment, you are given 100 points. These are yours just for agreeing to do the assignment. However, to keep them you will have to turn in an allocation for each of the classes for which this assignment continues. These points will be added to your total score on the final. Without the points you earn in this experiment, there would be 600 total points on the final.

For the first assignment, please allocate your points between two investments:

Investment #1, this is a risk-less investment, with a 0% return. You earn no points from this, but you keep the points you allocate to this investment.

Investment #2, Under this investment, you have a 50% chance of earning a 10% return on the points you allocate to this investment, and you have a 50% chance of earning a -5% return on the amounts allocated. That is if this goes the wrong way, you will lose 5% of the points allocated to this investment. This return will be determined by a coin flip that will be conducted next class. The coin flip will determine the returns for all of the students.

You can allocate anything from 0 to 100 points to either investment, you must allocate your entire 100 points, and you cannot allocate more than 100 points in total.

The allocation will be put on index cards which will identify the student by student number. This allocation will be collected at the beginning of the second class. I will keep ruling totals for each student. Each class I will hand out a sheet with all the returns of all students. I will post the final returns on the TWEN site so you will be able to know how many points are to be added to final.

Assignment 2

For this second assignment, you can allocate your points between two investments.

Investment #1, this is a risk-less investment, with a 0% return. You earn no points from this, but you keep the points you allocate to this investment.

Investment #2, Under this investment, you have a 50% chance of earning a 20% return on the points you allocate to this investment, and you have a 50% chance of earning a -12% return on the amounts allocated. That is if this goes
the wrong way, you will lose 12% of the points allocated to this investment. This return will be determined by a coin flip that will be conducted next class. The coin flip will determine the returns for all of the students.

You can allocate anything from 0 to 100 percent of your points to either investment, you must allocate all of your points, and you cannot allocate more than the number points you have. If you find it convenient, you can state your points in percentage terms, or you can state in point terms.

Assignment 3

For this third assignment, you can allocate your points between two investments.

Investment #1, this is a risk-less investment, with a 0% return. You earn no points from this, but you keep the points you allocate to this investment.

Investment #2, Under this investment, you have a 50% chance of earning a 28% return on the points you allocate to this investment, and you have a 50% chance of earning a -20% return on the amounts allocated.

Again you can state your allocation in percentage or point terms, whichever is more convenient to you.

Assignment 4

For this fourth assignment, you can allocate your points between two investments.

Investment #1, this is a risk-less investment, with a 0% return. You earn no points from this, but you keep the points you allocate to this investment.

Investment #2, Under this investment, you have a 50% chance of earning a 28% return on the points you allocate to this investment, and you have a 50% chance of earning a -20% return on the amounts allocated. Unlike the returns
before, the amount you earn on this investment will be subject to a 30% percent tax. Under this tax, if you earn 10 points, you will pay a tax of 3 points, and you get to keep 7 points. A symmetrical result obtains if you lose 10 points.

1 This Domar-Musgrave model makes certain assumptions about investment behavior. It essentially adopts the "mean-variance" assumption. Under this assumption, the only two characteristics of an investment that matter are the investment's expected value (the higher the better) and the variance (the lower the better). It is of course possible that other things also matter to the investor or that these factors matter but it manner different that accounted for in standard portfolio models. To the extent that this is the case, the Domar-Musgrave model will not capture the effects of these aspects of investment decisions.

2 Notice that, because there is no return on the riskless asset, this is a tax on all of the investor's income.

3 Proof:
If the final random wealth is $W = W_0 + (1-t)xr$, the individual (assuming they behave according to expected utility theory and are risk averse)

will try to maximize their expected utility or $E[U(W)] = \int_{-\infty}^{r^+} U(W_0 + (1-t)xr)f(r)dr$ to $\frac{\partial}{\partial x} E[U(W)]$, $E[U'(W)] = 0$, so

$\int_{-\infty}^{r^+} rU'(W_0 + (1-t)xr)f(r)dr = 0$ (and, simple differentiation by $x$ yields) $\int_{-\infty}^{r^+} (1-t)rU'(W_0 + (1-t)xr)f(r)dr = 0$ and, noting that dividing each side by $(1-t)$ will yield the result above.

4 Terrence R. Chorvat, Apologia for the Double Taxation of Corporate Income


7 Martin Feldstein, Personal Taxation and Portfolio Composition: An Econometric Analysis 44 Econometrica 631 (1976), there are also studies in other countries which seem to show this effect, se Jonas Agell and Pers-Anders Edin, Marginal Taxation and the Asset Returns of Portfolios of Swedish Households 92 Scandinavian J. Econ. 47, 62-3 (1992), and Stephan Hochguertel et al. Savings Accounts Versus Stcoks and Bonds in Household Portfolio Allocation 99 Scan. J. Econ. 81, 92 (1997). However, the results of all of these studies can also explained in other ways, perhaps more naturally in the case of the Feldstein study. Therefore, one cannot say that the econometric evidence behind the use of this model is strong.

8 See e.g., IRC § 1211

9 These are discussed in Stiglitz (1969).

10 See Sandmo The Effects of Taxation on Savings and Investment, 1 Handbook of Public Economics 265 (Auerbach and Feldstein eds., 1986)

11 Here, using a one tailed test for this case, we can state at the 95% confidence level that the mean amount allocated to the risky asset will not decrease by more than 14.8%.